Journal of Power Sources, *47* (1993) 159–175

Analysis of power and energy for fuel cell systems

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(Received March 8, 1993; accepted in revised form July 26, 1993)

Abstract

The relationship between power and energy for a fuel cell system consisting of a fuel cell stack and reactant storage subsystem, and operating at constant power or variable power, was analyzed. The characteristic parameters of the fuel cell stack and the reactant subsystem are considered to be independent variables, which are functions of the power and energy of the fuel cell system, respectively. Mathematical expressions were derived for determining the minimum weight of the fuel cell system when the cells operate at constant power and the cell voltage varies linearly with the current density. The relationship between the weight of a fuel cell system and variable power levels was also determined. These mathematical models were used to analyze the experimental results reported in the literature for an alkaline fuel cell and a polymer electrolyte fuel cell.

Introduction

A fuel cell that is used for utility power generation may generate electricity at constant power, but one used in portable applications and as a power source in electric vehicles is likely to operate periodically at several different power levels. In the case of fuel cells in electric vehicles, the power is likely to vary over a wide range during idling, cruising, and acceleration. It is critical that the fuel cell system be optimized for weight and performance in both electric vehicle and portable applications. A derivation of the power–energy relationship for a fuel cell system that includes the fuel cell stack, reactant and its storage containers, and the required connection plumbing between the two subsystems is presented. This derivation follows from the analysis and discussion presented by Weaver and Smith [1], Van Winkle and Carson [2], and Austin [3] on the optimization of fuel cell systems. The total weight, W_t, of a fuel cell system is considered to consist of the weight of the fuel cell stack, W_c, and the reactant (i.e., fuel and oxidant) subsystem, W_f, which also includes the necessary auxiliaries such as piping, plumbing, tanks, etc. The analysis of the system weight first considers operation of the fuel cell at constant power. This analysis is then expanded to consider the case of variable power, and is an extension of the work reported by Van Winkle and Carson [2].

The mathematical expressions that were derived for the performance (current, voltage, power, energy) and physical (weight of fuel cell stack and reactant subsystem) characteristics of fuel cells were entered into the Excel software package (Version 4.0, Microsoft) and used to analyze experimental data reported for two fuel cell systems.

Mathematical analysis

The single-cell voltage (V)–current density (i) relationship is assumed to be linear:

$$V = V_o - iR \tag{1}$$

where V_o is the open-circuit voltage obtained by extrapolation of the linear V–i plot to $i=0$, and R is the ohmic resistance of the cell (Ω cm^2), obtained from the slope of the V–i plot in the linear region. The weight of the fuel cell stack, W_c, is assumed to be a function of the power P delivered to the load:

$$W_c = \frac{Pp}{iV} \tag{2}$$

where p is a weight factor for the stack (weight/electrode area). The factor p can be related to the weight and total area of the electrodes in the fuel cell stack. Similarly, the weight of the reactant subsystem, W_f, is assumed to be a function of the energy, E, delivered by the fuel cell stack:

$$W_f = \frac{Ee}{\mu FV} \tag{3}$$

where F is Faraday's constant, μ an energy conversion efficiency (see below), and e a factor that is a function of the amount of reactants (number of equivalents) and the weight of the auxiliary components such as storage tanks, piping, etc. (weight/equivalent of stored energy). It is apparent from eqn. (3) that the weight of the reactant subsystem decreases with an increase in the energy conversion efficiency, which has a value that can range, in principle, from 0 to 1. Factors that can lead to a lower energy conversion efficiency include reactant leaks, cross-cover, parasitic chemical reactions, electrochemical reactions that only proceed to intermediate species, and partial utilization of the reactants.

Constant power

If the power output is constant with time (t), then:

$$E = Pt \tag{4}$$

and upon substituting in eqn. (3) yields:

$$W_f = \frac{Pte}{\mu FV} \tag{5}$$

A power–energy relationship can be derived for the fuel cell system which is based on the above analysis. Substituting eqn. (1) into eqn. (2) yields:

$$W_c = \frac{PRp}{V(V_o - V)} \tag{6}$$

The specific power of the fuel cell stack (P_s) is given by:

$$P_s = \frac{P}{W_c} = \frac{V(V_o - V)}{Rp} \tag{7}$$

Similarly, the specific energy of the reactant subsystem (E_s) is defined as:

$$E_s = \frac{E}{W_f} = \frac{\mu FV}{e} \tag{8}$$

and, solving for V yields:

$$V = \frac{E_s e}{\mu F} \tag{9}$$

Substituting for V in eqn. (7) gives:

$$P_s = \frac{E_s e}{\mu F R p}\left[V_o - \frac{E_s e}{\mu F}\right] \tag{10}$$

$$= \frac{V_o^2}{Rp}\left[\frac{E_s e}{\mu F V_o} - \frac{E_s^2 e^2}{\mu^2 F^2 V_o^2}\right] \tag{11}$$

We can define:

$$Q_o = \mu F \tag{12}$$

where Q_o is the maximum available capacity of the reactant, and is equal to Faraday's constant (26.8 Ah/eq) when $\mu = 1$. Following this argument, it is assumed that $Q_o V_o$ is the maximum electrical energy that can be obtained by electrochemical conversion of the reactant. Substituting eqn. (12) into eqn. (11) gives:

$$P_s = \frac{V_o^2}{Rp}\left[\frac{E_s e}{Q_o V_o} - \frac{E_s^2 e^2}{Q_o^2 V_o^2}\right] \tag{13}$$

It is interesting to note that the form of this equation is similar to that for the energy–power relationship for batteries which was derived by McLarnon *et al.* (4), as noted in the following equation:

$$P_s = \frac{V_o^2}{R}\left[\frac{E_s^{1/2}}{Q_o^{1/2} V_o^{1/2}} - \frac{E_s}{Q_o V_o}\right] \tag{14}$$

Comparison of eqns. (13) and (14) shows that the exponents for E_s are different. In addition, the factors (i.e., p, e) related to the characteristics of the fuel cell stack and the reactant system are also included in eqn. (13) because the power and energy of the fuel cell system are varied independently.

A schematic representation of the relationship between the dimensionless voltage V/V_o and the weight of the fuel cell stack and reactant subsystems is presented in Fig. 1 along with the total system weight. The relationship between the voltage and the minimum weight of a fuel cell stack operating at constant power is obtained by solving for V in eqn. (6) when $dW_c/dV = 0$; this occurs at $V = 0.5V_o$. On the other hand, the weight of the reactant subsystem decreases with an increase in V/V_o, and no minimum weight is obtained. The total fuel cell system weight is equal to the weight of the fuel cell stack and reactant subsystem. It exhibit a minimum weight when $V/V_o \geqslant 0.5$ (~ 0.6 for the example shown in Fig. 1).

The minimum weight of the fuel cell system operating at constant output power can be obtained as a function of stack voltage. Using:

$$W_t = W_c + W_f \tag{15}$$

and substituting eqns. (5) and (6), we obtain:

$$W_t = \frac{PRp}{V(V_o - V)} + \frac{Pte}{\mu F V} \tag{16}$$

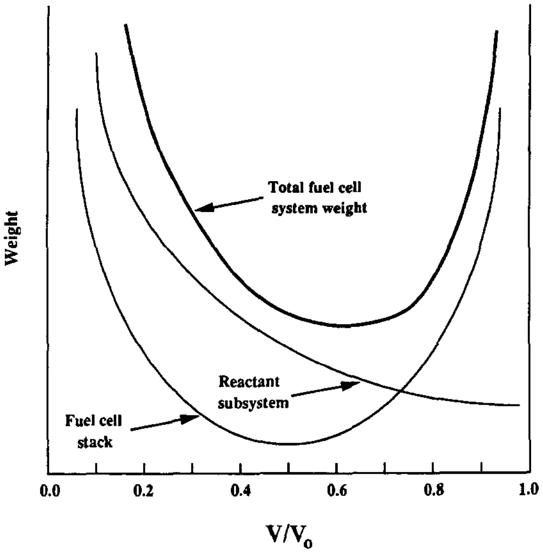


Fig. 1. Schematic representation of the relationship between the cell voltage ratio V/V_0 and the weights of the total fuel cell system, fuel cell stack and reactant subsystem.

or:

$$W_t = \frac{P}{V}\left[\frac{Rp}{(V_0 - V)} + \frac{te}{\mu F}\right] \tag{17}$$

Following the analysis by Van Winkle and Carson [2], the stack voltage for the minimum weight of the fuel cell system is obtained by solving for V from eqn. (17) when $\mathrm{d}W_t/\mathrm{d}V = 0$:

$$\frac{V}{V_0} = \frac{X + 1 - (X+1)^{1/2}}{X} \tag{18}$$

where $X = \dfrac{V_0 t}{\mu F R}\dfrac{e}{p}$. For large t (i.e., $t \gg o$), V approaches V_0, and the fuel cell should be operating at close to the open-circuit voltage for minimum weight of the system. The plot in Fig. 2 shows the relationship between V/V_0 and X. It is apparent that X approaches $V = 0.5\ V_0$ as X becomes very small, and $V = V_0$ as X becomes very large. The net result is that, regardless of the value of X, the mininum weight is obtained at $V \geqslant 0.5V_0$.

Optimizing the weight of the fuel cell system is important for fuel cells that are used in portable, transportation, space, and underwater applications. Furthermore, optimizing in terms of the cell voltage is useful because the efficiency of the fuel cell (ϵ_{FC}) is proportional to the cell voltage [5], i.e.:

$$\epsilon_{FC} = \frac{nF\mu V}{\Delta H_c} \tag{19}$$

where ΔH_c is the enthalpy change for the combustion of the fuel.

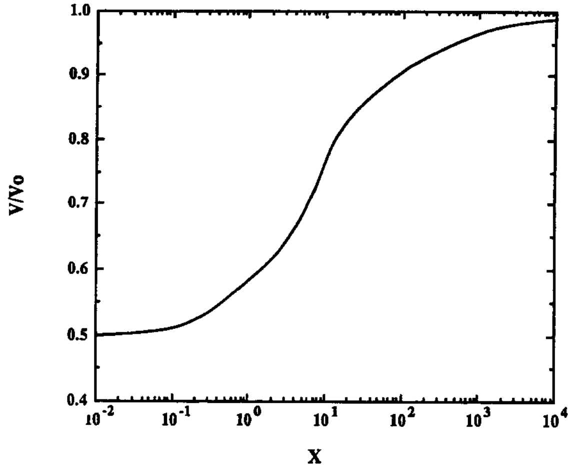


Fig. 2. Relationship between dimensionless parameter X and the cell voltage ratio V/V_o.

Substituting for V from eqn. (18) into eqn. (19) gives:

$$\epsilon_{FC}=\frac{nF\mu V_o}{\delta H_c}\left[\frac{X+1-(X+1)^{1/2}}{X}\right] \tag{20}$$

For hydrogen, $\Delta H_c/nF=1.481$ V (higher heating value), and substituting in eqn. (20) yields:

$$\epsilon_{FC}=\frac{\mu V_o}{1.481}\left[\frac{X+1-(X+1)^{1/2}}{X}\right] \tag{21}$$

Assuming that $\mu=100\%$, the relationship between the cell voltage and efficiency at which the minimum weight of a H_2/O_2 fuel cell is obtained is:

$$V=1.481\epsilon_{FC} \tag{22}$$

The efficiency and dimensionless voltage exhibit the same general trends as a function of X because they both show the same dependence on X, i.e., the efficiency and X increase with an increase in V.

The operating time of the fuel cell system has a strong influence on the optimum weight of the system. This point can be illustrated by the relationship that is obtained by combining eqns. (4), (7), (8) and (15):

$$W_t=\frac{E}{tP_s}+\frac{E}{E_s} \tag{23}$$

From eqn. (23), it is apparent that W_t will increase as the operating time of the fuel cell decreases (with $\mu=1$), when the other characteristic parameters are constant. Under these conditions, X decreases with a decrease in t, and the cell voltage also decreases.

Variable power

Van Winkle and Carson [2] have also considered variable power levels and nonlinear V–i plots in their analysis of fuel cell systems. They provided figures-of-merit which characterize the physical parameters related to the fuel cell stack (f_c)

and the reactant subsystem (f_f). The value of f_c increases as the specific power increases, and f_f increases as the mass of the reactant subsystem increases. These parameters provide useful insight for optimizing a fuel cell system, and complement the information gained by analyzing the power–energy plots. The analysis presented by Van Winkle and Carson [2] is used as the basis for the following discussion.

The power output varies with a change in the cell voltage and current. When the voltage, V_1, changes to another voltage, V_i, the power can change from P_1 to P_i. The relationship between power and the electrochemical parameters, current and cell voltage, is defined as:

$$P = VI \tag{24}$$

Combining the linear current–voltage relationship given by eqn. (1) and eqn. (24) yields:

$$A = \frac{PR}{V(V_o - V)} \tag{25}$$

where A is the electrode area. For two power levels, it is easy to see that:

$$\frac{P_1 R}{V_1(V_o - V_1)} = \frac{P_i R}{V_i(V_o - V_i)} \tag{26}$$

when the electrode area is the same for both cases. Solving the quadratic equation for V_i yields:

$$V_i = \frac{V_o + [V_o^2 - 4(P_i/P_1)V_1(V_o - V_1)]^{1/2}}{2} \tag{27}$$

For variable power levels, eqn. (16) for the weight of the fuel cell system is modified to:

$$W_t = \frac{P_1}{V_1}\left[\frac{Rp}{(V_o - V_1)} + \frac{\mathrm{e}}{\mu \mathrm{F}}\frac{\mathrm{V}_1}{P_1}\Sigma\frac{\mathrm{P}_1 t_i}{V_i}\right] \tag{28}$$

In the analysis by Van Winkle and Carson [2], they selected P_1 equal to the 'largest power output to be delivered by the system' which means that the fuel cell system must be capable for continuous operation at a power level P_1. In the following discussion P_1 is set equal to $P_{\max}$, which is the maximum power and occurs at $V_1 = \mathrm{V}_o/2$ for a cell operating with a linear current–voltage relationship. In this case, $P_{\max}$ is defined as:

$$P_{\max} = (V_o^2/4R)A_t \tag{29}$$

where A_t is the total electrode area in the fuel cell stack. Making the substitution for P_1 in eqn. (28) yields:

$$W_t = \frac{4RpP_{\max}}{V_o^2} + \frac{2e}{\mu F V_o}\Sigma\left[\frac{P_i t_i}{1 + [(P_{\max} - P_i)/P_{\max}]^{1/2}}\right] \tag{30}$$

The amount of fuel that is required is a strong function of the operating cell voltage of the fuel cell. For the case of hydrogen oxidation, the coulombic charge that is available is 53.6 Ah/mol or 26.8 Ah/g H_2 (Faraday's law). The weight of H_2 that is required to produce energy E can be computed from the equation:

$$W_{H_2}(g) = \frac{E}{26.8(\mathrm{Ah/g})V} = \frac{Pt}{26.8(\mathrm{Ah/g})V} \tag{31}$$

where V is the cell voltage. If we assume that the i–V relationship is linear (see eqn. (1)), then W_{H_2} can be represented as a function of P by solving for V. The power, P, can be expressed as a function of current and voltage, i.e.:

$$P = VI = ViA_t \tag{32}$$

Substituting for i from eqn. (1) in eqn. (32) yields:

$$P = \frac{VA_t(V_o - V)}{R} \tag{33}$$

and solving for V gives:

$$V = \frac{V_o}{2} \pm \frac{1}{2}\left[V_o^2 - \frac{4PR}{A_t}\right]^{1/2} \tag{34}$$

Only the positive root $V > V_o/2$ will be considered because in normal fuel cell operation V is greater than the value corresponding to that at maximum power. Substituting eqn. (34) in eqn. (31) yields:

$$W_{H_2}(g) = \frac{Pt}{26.8}\left[V_o + \left(V_o^2 - \frac{4PR}{A_t}\right)^{1/2}\right]^{-1} \tag{35}$$

or substituting for V_o from eqn. (29) and considering the case of variable power P_i we arrive at:

$$W_{H_2}(g) = \frac{2}{26.8V_o}\Sigma\left[\frac{P_i t_i}{1 + [(P_{max} - P_i)/P_{max}]^{1/2}}\right] \tag{36}$$

Description of fuel cell systems

Examples that illustrate the application of the energy–power relationship (eqn. (13)) for a fuel cell system are presented below, using data from Rosso *et al.* [6] for a polymer electrolyte fuel cell (PEFC) and Wyczalek and coworkers [7, 8] for an alkaline fuel cell (AFC). The PEFC was designed for use in a space station, while the AFC was intended for use in a van built by the General Motors Corporation. The specifications for these system are presented in Table 1, and additional experimental performance parameters for PEFC are given in Table 2.

The PEFC system contains a fuel cell stack with 32 bipolar cells operated in a 'dead ended' mode. In this fuel cell system, the H_2 is stored as a metal hydride in six 3.2-cm diameter stainless-steel containers. The fuel cell stack is thermally integrated with the H_2 storage containers by thermal conduction through metal plates. It is reported that about 25% of the available waste heat is utilized for releasing H_2 from the metal hydride. The capacity of H_2 that is stored is equivalent to 33.8 Ah/cell. However, the H_2 utilization decreases as the current increases because the amount of H_2 that is available from the hydride decreases. The factor μ is assumed to be equal to the utilization of the H_2, and it is listed in Table 2.

The AFC power plant and the reactant subsystem were designed to demonstrate the viability of fuel cell power plants in transportation applications. The fuel cell stack, which consumed pure H_2 and O_2, consisted of 32 modules (17 cells each) that were connected electrically in series. The KOH electrolyte was circulated through the fuel

TABLE 1

Specifications for PEFC for space applications and AFC for electric vehicles

	PEFC [6]	AFC [7, 8]
Fuel cell systems		
Volume (l)	6.8	2208.7
Weight (kg)	15.9	1534.5
Power rating (kW)	0.196	32
Energy rating (kWh)	0.952	123[a]
Fuel cell stack		
Capacity/cell (Ah)	34	268
Voltage (V)	28 ± 4	460
Current rating (A)	7	69
Number of modules	1	32
Number of cells	32	544
Electrode area (cm^2)	70	1282
Weight (kg)	10	1190
Reactant subsystem		
Weight (kg)	5.9	344.5
H_2 storage	metal hydride	liquid
Volume H_2 (24 °C, 1 atm) (l)	485	
Moles H_2	19.8	2724
Oxygen storage	compressed gas	liquid
Parameters		
p (g/cm^2)	4.46	1.71
e (g/eq)	150	63.25
R (Ω cm^2)	0.56	0.65
V_o (V)	0.92	0.90

[a]Capacity/cell X voltage.

TABLE 2

Experimental performance parameters for PEFC [6]

Stack current (A)	Stack voltage (V)	Capacity (Ah)	Specific[a] power (W/kg)	Specific[b] energy (Wh/kg)	μ^c
7	27.6	33.4	19.4	156.2	0.988
10	26.4	33.3	26.4	149.0	0.985
11.5	26.2	31.6	30.3	140.3	0.935
13.8	26.0	26.7	35.8	117.7	0.790
15.9	25.5	24.4	40.5	105.5	0.722

[a]Weight of fuel cell stack = 10 kg.
[b]Weight of reactant subsystem = 5.9 kg.
[c]$\mu = 1$ for capacity of 33.8 Ah H_2.

cell stack by three magnetically-driven pumps to remove waste heat and transport gases out of the modules. The electrolyte temperature was maintained below 66 °C by dissipating heat from a nickel heat exchanger. Hydrogen and O_2 were stored cryogenically to minimize the weight and volume of the reactant subsystem, although high-pressure gas storage was also considered.

Analysis of fuel cells operating at constant power

Polymer electrolyte fuel cell

The results obtained from the mathematical analysis presented above were compared with the reported experimental data (see Tables 1 and 2). The cell voltage at a current density of 0.1 A/cm^2 (7 A) is 0.863 V, in good agreement with the value (0.864 V) determined by eqn. (1) and (34) with $P=194$ W. Using this computed cell voltage and $P=194$ W in eqn. (31) yields a weight of H_2 of 39.8 g or equivalent to 33.3 Ah, which is in good agreement with the measured value of 33.4 Ah. Reasonably good agreement between the measured and calculated weights of H_2 was obtained for the other experimental results in Table 2.

A plot of the specific power of the fuel cell stack as a function of the specific energy of the reactant subsystem is presented in Fig. 3. This form of data presentation is often referred to as a Ragone plot and is commonly used to assess the performance characteristics of batteries. Eqn. (13) and the parameters listed in Table 1 were used to obtain the calculated values in Fig. 3, and they are represented as solid lines. Plots of specific power versus specific energy for values of $\mu=1.0$, 0.9, 0.8 and 0.7 are presented. The solid symbols represent the experimental data listed in Table 2, with their respective values of μ indicated in the Fig. A remarkably close agreement is obtained between the calculated and experimental values, when the factor μ is considered in the analysis.

Alkaline fuel cell

The relationship between cell voltage and power obtained from eqn. (34) is presented in Fig. 4 for the AFC which is described in Table 1. The cell voltage decreases dramatically at high power, particularly near the maximum power where $V=V_0/2$ (i.e., 0.45 V). At the rated power output of 32 kW, the cell voltage is calculated to be 0.86 V, in agreement with the value reported by Wyczalek *et al.* [8]. The maximum

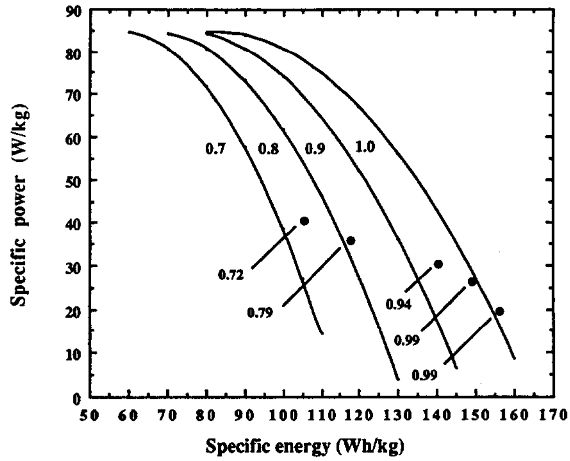


Fig. 3. Power–energy relationships for PEFC.

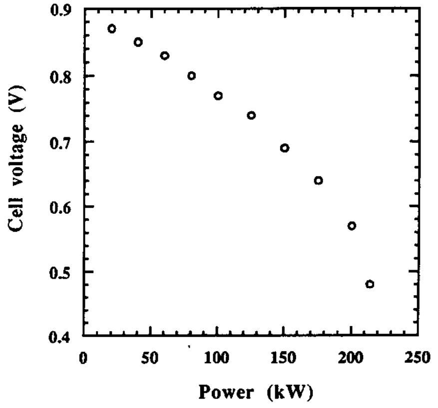


Fig. 4. Plot of cell voltage of an AFC as a function of power output.

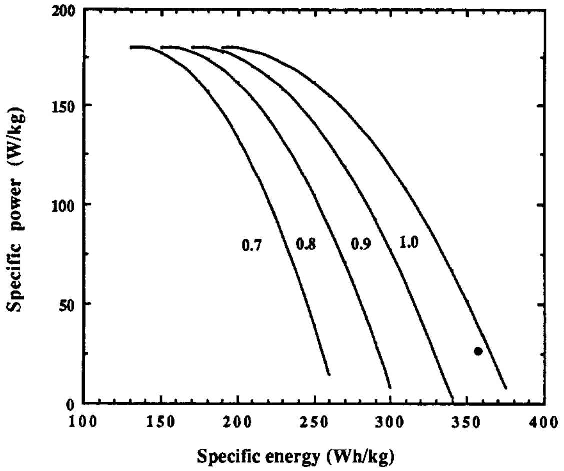


Fig. 5. Power–energy relationships for AFC.

power that is calculated for this fuel cell from eqn. (29) is 217 kW. This value agrees reasonable well with the data reported by Wyczalek *et al.* (Fig. 17 in ref. 8) where each module achieved about 6.2 kW, or 198 kW for a 32-module stack.

The specific power of the fuel cell stack is plotted as a function of the specific energy of the reactant subsystem in Fig. 5. The characteristic features of these curves are very similar to those shown in Fig. 3, except the range of specific energies and specific power are higher. On the basis of the experimental data in Table 1, the calculated values for the AFC system are: $E_s = 357$ Wh/kg and $P_s = 27$ W/kg, which is indicated by the solid symbol in Fig. 5. This calculated value lies very close to the curve where $\mu = 1.0$.

The validity of eqn. (31) to predict the amount of H_2 consumed in the AFC was examined. Wyczalek *et al.* [8] reported data on the amount of H_2 consumed as a function of power output by the fuel cell. When the fuel cell is idling, it requires 5 kW to operate the ancillaries and consumes 0.5 lb H_2/h. From eqn. (31), we calculate $W_{H_2} = 0.43$ lb H_2/h, in reasonable agreement with the reported value. At the rated power of 32 kW, the H_2 consumption rate is 3.2 lb H_2/h. In this case, the value from eqn. (31) is 3.1 lb H_2/h, which is in good agreement with the reported value. Based on this simple comparison, it appears that eqn. (31) is useful for predicting the rate of H_2 consumption in the AFC that was tested by Wyczalek *et al.* [8].

Optimization of fuel cell system

The relationship between V/V_o and X (eqn. (18)) is useful for determining the cell voltage at which the minimum weight of the fuel cell system is obtained. The values of X for the PEFC, which range from about 4 to 10, were derived from the data in Tables 1 and 2. The AFC is projected to consume 1.45 kg/h H_2, and with 5.45 kg storage, the operating time is 3.75 h when $\mu=1$. With $t=3.75$ h, $\mu=1$, and the AFC parameters listed in Table 1, $X=7.166$ is obtained. Substituting for X in eqn. (18) yields, $V/V_o=0.74$ for the AFC system, and $V=0.67$ V when $V_o=0.90$ V. Figure 6 shows plots of W_c, W_f and W_t as a function of cell voltage for the AFC which is described in Table 1. These results show that the minimum weight of the fuel cell system is obtained at a cell voltage of 0.67 V, and that W_c and W_t increase rapidly at higher cell voltages. Similar plots (see Fig. 7) to those in Fig. 6 are obtained for the PEFC described in Table 1. In this case, the minimum system weight is observed at ~0.7 V for $\mu=1$. The results in Figs. 6 and 7 show that the minimum weight of the fuel cell stack occurs at $V=0.5$ V_o, as expected from differentiation of eqn. (6). When the values for W_c, W_f and W_t in Table 1 are matched to the cell voltages in Figs. 6 and 7, it is apparent that neither fuel cell systems was optimized for minimum

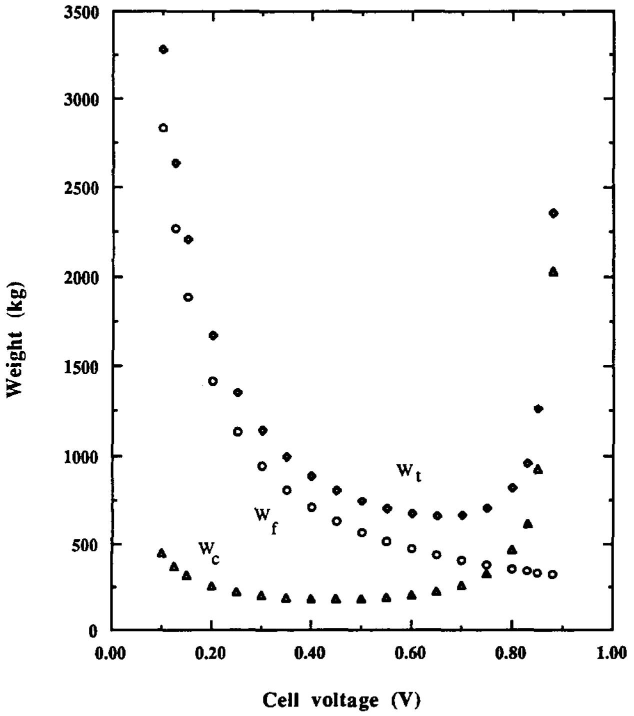


Fig. 6. Analysis of cell voltage and optimum weight of AFC.

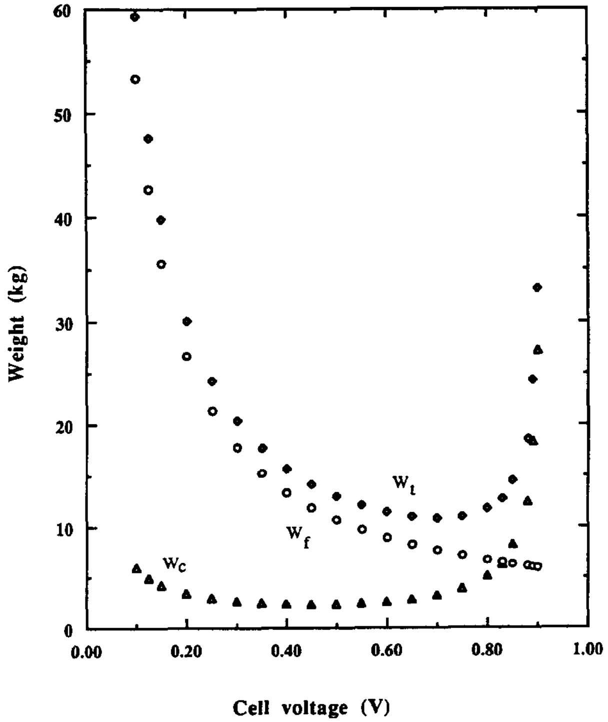


Fig. 7. Analysis of cell voltage and optimum weight of PEFC.

weight. For both the AFC and PEFC, the cell voltages which correspond to the values of W_c, W_f, and W_t in Table 1 are in the range from 0.8 to 0.9 V.

Analysis of fuel cell operating at variable power

A fuel cell with a fixed amount of H_2 storage will have operating times that vary considerably at different power outputs, as evident in the relationship between the amount of H_2, power, and cell voltage (see eqn. (31)). This point is illustrated by the results in Fig. 8, which show that the operating time of an AFC (specifications in Table 1) with 500 g of H_2 available decreases with an increase in power output. If the AFC operates continuously at its rated power of 32 kW, this amount of H_2 will last for about 0.3 h. However, if the fuel cell operates continuously near maximum power, the 500 g H_2 will be consumed in less than 0.03 h. This analysis can be extended further to determine the amount of H_2 that is consumed by the AFC when it generates a specific amount of electrical energy, 12 kWh for example, at different power outputs. The results are presented in Fig. 9. As expected, there is a dramatic difference in the amount of H_2 that is required to generate 12 kWh at the rated power and at higher power levels. In fact there is a nearly two-fold increase in the H_2 consumption by the AFC when it operates near the maximum power level. Correspondingly, a lesser amount of H_2 is required if the fuel cell operates at power levels less than the rated power. These observations have significant implications for a fuel cell that operates at variable power, particularly at higher power.

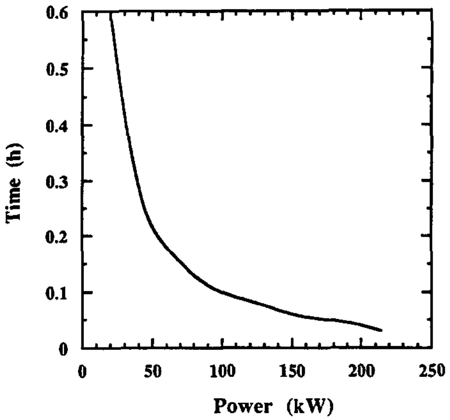


Fig. 8. Operating time at different power levels for fuel cell with 500 g H_2.

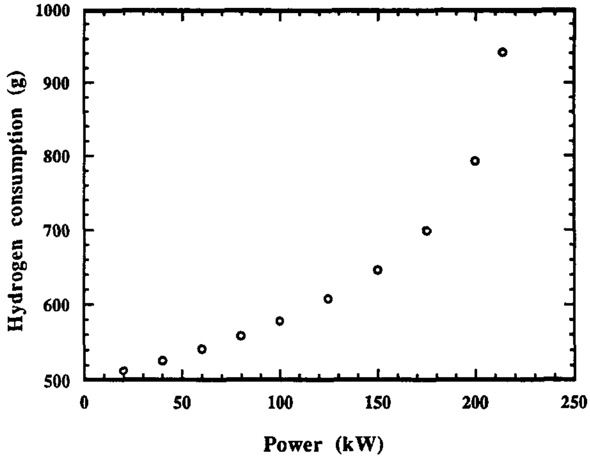


Fig. 9. Hydrogen consumption in fuel cell that generates 12 kWh at different power outputs.

A fuel cell system that is used in transportation applications, such as electric vehicles, will contain a fixed amount of H_2. The total energy that is generated will vary with the peak power and will be limited by the amount of H_2 that is carried on-board the vehicle. To the best of our knowledge, there are no standard tests to assess the performance of fuel cells for electric vehicle applications. Therefore, we have chosen to adapt the Generic Simplified Federal Urban Driving Schedule (GSFUDS) test cycle to evaluate the performance of fuel cells for electric vehicle applications. The GSFUDS is a power profile test that utilizes specific power and time segments to evaluate the performance of batteries for electric vehicle applications [9]. The GSFUDS test cycle is not dependent upon specific vehicle characteristics, and it is described with specific power levels that are defined as integer multiples of the average power (P/P_{ave}). The P–t profile for the GSFUDS test cycle includes periods when the battery is being charged, which simulates 'regenerative braking' of the electric vehicle. This part of the P–t profile is not included in the following evaluation of the AFC, because the fuel cell is only operated to generate electrical power. Figure 10 shows a schematic representation of the GSFUDS test cycle without the segments for regenerating braking and idling (see also Table 3).

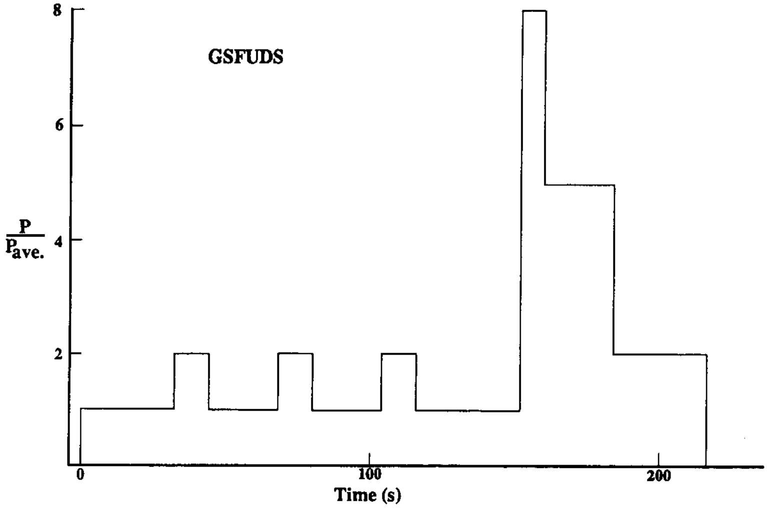


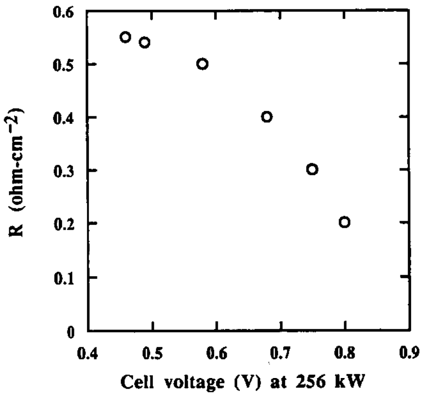
Fig. 10. Modified GSFUDS test cycle.

TABLE 3

Power–time profile for modified GSFUDS test cycle

P/P_{ave}	Power (kW)	Time (s)
1	32	28
2	64	12
1	32	24
2	64	12
1	32	24
2	64	12
1	32	36
8	256	8
5	160	24
2	64	32
Total		212

In the case of the AFC described in Table 1, the average power output is assumed to be equal to the rated power of 32 kW or 21 W/kg. This fuel cell stack is capable of achieving a maximum power of about 200 kW, which is less than 8 times the average power (i.e., 256 kW), and therefore it cannot meet the peak power requirement established for the GSFUDS test cycle. From the preceding mathematical expressions, we can ascertain what change in physical parameter is required for the AFC to meet a peak power of 256 kW for the GSFUDS test cycle. For instance a decrease in the cell resistance will be beneficial in increasing the peak power, which is attained at $V \geqslant 0.5V_0$ (i.e. $\geqslant 0.45$ V). The cell resistance was varied between 0.65 and 0.1 Ω cm^2 and the cell voltage was computed from eqn. (34). The results in Fig. 11 show that cell voltages $\geqslant 0.45$ V are obtained at cell resistances less that about 0.55 Ω cm^2, and



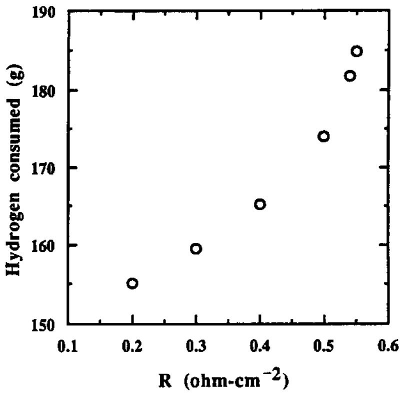


Fig. 11. Influence of cell resistance on the cell voltage at a peak power of 256 kW.

Fig. 12. Hydrogen consumption during modified GSFUDS test cycle, assuming a cell resistance that varies from 0.1 to 0.55 Ω cm^2.

the cell voltage at a peak power of 256 kW increases dramatically when the cell resistance is decreased to 0.1 Ω cm^2.

The amount of H_2 that is consumed during each GSFUDS test cycle with an AFC (see Table 1) and assuming that the cell resistance is reduced to permit a peak power of 256 kW, is calculated from eqn. (36) and plotted in Fig. 12. The amount of H_2 that is required in each GSFUDS test cycle increases with an increase in cell resistance and a decrease in cell voltage. Furthermore, the amount of H_2 that is required increases with an increase in the peak power, and its shows a close correlation with the increase in the weight of the system. This is evident by substituting eqn. (36) in eqn. (30) which yields:

$$W_t = \frac{4RpP_{\max}}{V_o^2} + eW_{H_2} \qquad (37)$$

The system weight increases linearly with an increase in the amount of H_2 that is required, assuming that the factor e is a constant. It is likely that the factor e will not be constant because components such as piping, heat exchangers, pumps, valves, etc., may be insensitive to the amount of H_2 that is stored and the operating characteristics of the fuel cell. On the other hand, the weight and size of the storage container are dependent on the amount of H_2. It is evident that the weight of a fuel cell system that operates at variable power levels above the average power rating must be greater than one that operates at constant power (assuming both fuel cells generate the same total energy). In addition, the weight of the fuel cell system must be greater if it operates at higher power levels and generates the same amount of energy as one operating at lower power levels. The additional weight can be traced to the weight of the larger H_2 storage system that is needed because more H_2 is electrochemically oxidized when the cell operates at a lower cell voltage, or higher power output.

Concluding remarks

The analysis that is presented in instructive for determining the relationship between the electrochemical parameters and the optimum weight of a fuel cell that operates at constant power. The characteristic parameters of the fuel cell stack and the reactant subsystem are considered to be independent variables, which are functions of the power and energy of the fuel cell system, respectively. An energy–power relationship similar to the Ragone plots that are common to batteries were derived for fuel cells that operate at constant power, and this relationship shows good agreement with experimental data for both a PEFC and AFC. It should be noted that this good agreement was obtained for two specific fuel cells which are of limited power and size. However, these two fuel cell systems are of the power range for portable and electric vehicle applications, where the optimization of power and energy is most critical. Because a linear current–voltage curve is used in the analysis, the power output for which the analysis is valid will be limited. In the case of the PEFC data, the mathematical equations were useful for the analysis extending over a two-fold change in output power. Mathematical expressions were also derived for the relationship between the weight of a fuel cell system and variable power output. Several examples are provided to illustrate the relationships between the amount of fuel (H_2) consumed, operating time and variable power level.

Acknowledgement

This work was supported by the Assistant Secretary for Conservation and Renewable Energy, Office of Transportation Technologies, Electric/Hybrid Propulsion Division of the US Department of Energy under Contract No. DE-AC03-76SF00098. The authors would like to thank Frank McLarnon and Kathryn Striebel for their constructive comments and review of the manuscript.

List of symbols

A	electrode area in a cell, cm^2
A_t	total electrode area in fuel cell stack, cm^2
E	energy delivered by fuel cell stack, W
E_s	specific energy of reactant subsystem, Wh/kg
e	factor that is function of amount of reactants (number of equivalents) and weight of the auxiliary components such as storage tanks, piping, etc. (weight/equivalent of stored energy), g/eq
F	Faraday's constant, 26.8 Ah/eq
f_c	figure-of-merit for fuel cell stack
f_f	figure-of-merit for reactant subsystem
ΔH_c	enthalpy change for combustion of the fuel, cal/mol
I	current, A
i	current density, A/cm^2
n	number of electrons participating in electrochemical reaction
P	power, W
P_{ave}	average power, W
P_{max}	maximum power, W
P_s	specific power of fuel cell stack, W/kg
p	weight factor for stack (weight/electrode area), g/cm^3
Q_o	maximum available capacity of reactant, Ah
R	ohmic resistance of cell, $\Omega\ cm^2$

t	time, h
V	single-cell voltage, V
V_o	open-circuit voltage, V
W_c	weight of fuel cell stack, kg
W_f	weight of reactant (i.e., fuel and oxidant) subsystem, kg
W_{H_2}	weight of H_2, g
W_t	total weight of fuel cell system, kg
X	$\frac{V_o t}{\mu F R} \frac{e}{p}$
ϵ_{FC}	efficiency of fuel cell
μ	energy conversion efficiency

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